

Nuclear matter from chiral low-momentum interactions

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Nuclear matter calculations based on low-momentum interactions derived from chiral nucleon-nucleon and three-nucleon effective field theory interactions and fit only to few-body data predict realistic saturation properties with controlled uncertainties. This is promising for a unified description of nuclei and to develop a universal density functional based on low-momentum interactions.

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Nuclear forces saturate, so nuclei are self bound with roughly constant interior density. The Coulomb interaction drives heavier stable nuclei toward an imbalance of neutrons over protons and eventual instability, but theorists can extrapolate a Coulomb-free, $N = Z$ nucleus to arbitrary size. The uniform limit is called symmetric nuclear matter. For fifty years, an accurate prediction of nuclear matter starting from nuclear forces has been a theoretical milestone on the way to finite nuclei, but has proved to be an elusive target. Here we present the first nuclear matter calculations using soft Hamiltonians derived from chiral effective field theory interactions fit only to few-body ($A \leq 4$) data. We find realistic saturation properties with controlled uncertainties.

Despite the long-term emphasis on the infinite uniform system, most advances in microscopic nuclear structure theory over the last decade have been through expanding the reach of few-body calculations. This has unambiguously established the quantitative role of three-nucleon forces (3NF) for light nuclei ($A \leq 12$) [1, 2]. However, until now few-body fits have not sufficiently constrained 3NF contributions at higher density such that nuclear matter calculations are predictive. Nuclear matter saturation is very delicate, with the binding energy resulting from cancellations of much larger potential and kinetic energy contributions. When a quantitative reproduction of empirical saturation properties has been obtained, it was imposed by hand through adjusting short-range three-body forces (see, for example, Refs. [3, 4]).

Progress for controlled nuclear matter calculations has long been hindered by the difficulty of the nuclear many-body problem when conventional nuclear potentials are used. The present calculations pull together several recent developments to overcome the hurdles: systematic starting Hamiltonians based on chiral effective field theory (EFT) [5, 6], renormalization group (RG) methods [7, 8] to soften the short-range repulsion and short-range tensor components of the initial chiral interactions [9] so that convergence of many-body calculations is vastly accelerated [10, 11, 12], and an improved 3NF

fitting procedure to the ^4He radius [2]. In combination we obtain controlled theoretical uncertainties.

Our results are summarized in Fig. 1, which shows the energy per particle of symmetric matter as a function of Fermi momentum k_F , or the density $\rho = 2k_F^3/(3\pi^2)$. A grey square representing the empirical saturation point is shown in each of the nuclear matter figures. Its boundaries reflect the ranges of nuclear matter saturation properties predicted by phenomenological Skyrme energy functionals that most accurately reproduce properties of finite nuclei. Although this determination cannot be completely model independent, the value is generally accepted for benchmarking infinite matter. In the future, calculations of the properties of finite nuclei will allow one to compare directly to experimental data.

The calculations of Fig. 1 start from the $N^3\text{LO}$ nucleon-nucleon (NN) potential (EM 500 MeV) of Ref. [5]. This NN potential is RG-evolved to low-momentum interactions $V_{\text{low } k}$ with a smooth $n_{\text{exp}} = 4$ regulator [8]. For each cutoff Λ , two couplings that determine the shorter-range parts of the $N^2\text{LO}$ 3NF [13] are fit to the ^3H binding energy and the ^4He matter radius using exact Faddeev and Faddeev-Yakubovsky methods as in Ref. [14]. Our 3NF fit values are given in Table I. We use the same 3NF regulator $\exp[-(p^2 + 3/4q^2)/\Lambda_{3\text{NF}}^4]$ of Ref. [13], but with a 3N cutoff $\Lambda_{3\text{NF}}$ that is allowed to vary independently of the NN cutoff. The shaded regions in Fig. 1 show the range of results for $2.0\text{ fm}^{-1} < \Lambda_{3\text{NF}} < 2.5\text{ fm}^{-1}$ at fixed $\Lambda = 2.0\text{ fm}^{-1}$. Nuclear matter is calculated in three approximations: Hartree-Fock (left) and including approximate second-order (middle) and summing particle-particle-ladder contributions (right). The technical details are given in Ref. [10], but we have improved the calculation to include full momentum-dependent Hartree-Fock propagators and (sub-leading) 3N double-exchange diagrams beyond Hartree-Fock, however in a very approximate way. Further improvements are in progress [15].

The Hartree-Fock results show that nuclear matter is bound even at this simplest level. A calculation without

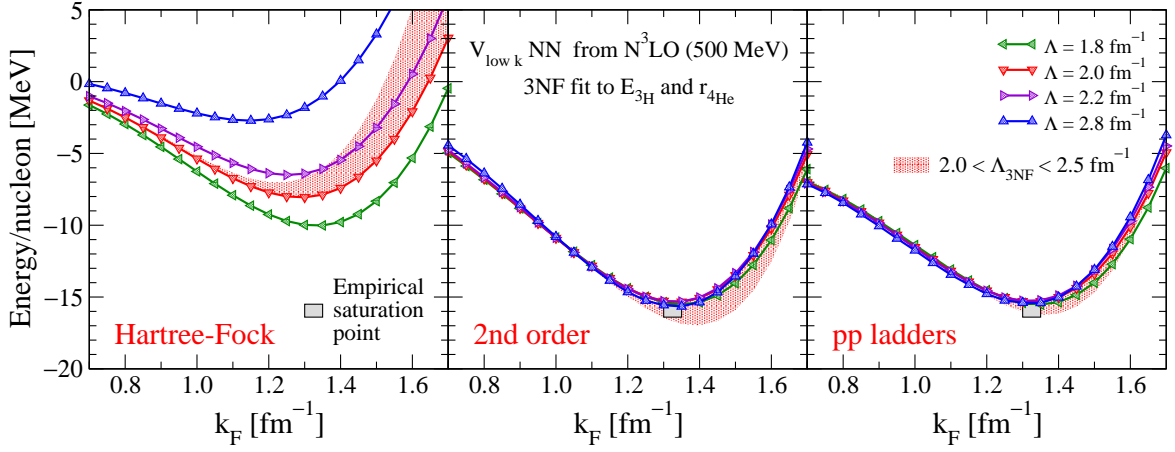
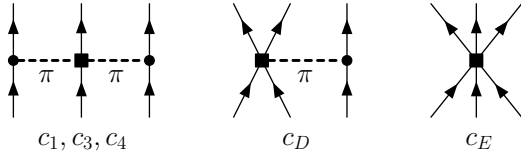


FIG. 1: (Color online) Nuclear matter energy per particle as a function of Fermi momentum k_F at the Hartree-Fock level (left) and including second-order (middle) and particle-particle-ladder contributions (right), based on evolved N^3 LO NN potentials and 3NF fit to E_{3H} and r_{4He} . Theoretical uncertainties are estimated by the NN (lines) and 3N (band) cutoff variations.

approximations should be independent of the cutoffs, so the spread in Fig. 1 sets the scale for omitted many-body contributions. The second-order results show a dramatic narrowing of this spread, with predicted saturation consistent with the empirical range. The narrowing happens across the full density range. This is strong evidence that these encouraging results are not fortuitous, but that we have reached cutoff independence at the level of 1–2 MeV per particle. The 3NF fits to the ^4He radius improve the cutoff independence significantly compared to fitting to $A = 3, 4$ energies only, see Fig. 6 in Ref. [10]. For all cases, the compressibility $K = 190 - 240$ MeV (mainly from $\Lambda_{3NF} = 2.0 - 2.5 \text{ fm}^{-1}$) is in the empirical range. To our knowledge, these are the first nuclear forces fit only to $A \leq 4$ nuclei that predict saturation reasonably.

The particle-particle-ladder sum is little changed from second order except at the lowest densities shown. This is not surprising because at low density the presence of a two-body bound state necessitates a nonperturbative summation. We note that below saturation density, the ground state of nuclear matter is not a uniform system, but breaks into clusters (see, for example, Ref. [16]).

In chiral EFT without explicit Deltas, 3N interactions start at N^2 LO [13] and their contributions are given diagrammatically by



We assume that the c_i coefficients of the long-range two-pion-exchange part are not modified by the RG and take these values from Ref. [17]. At present, we rely on the N^2 LO 3NF as a truncated “basis” for low-momentum 3N interactions and determine the c_D and c_E couplings by

| Λ or λ/Λ_{3NF} [fm^{-1}] | $V_{\text{low } k}$ | | SRG | |
|---|---------------------|---------|--------|---------|
| | c_D | c_E | c_D | c_E |
| 1.8/2.0 | -0.0112 | -0.2212 | | |
| 2.0/2.0 | -0.3000 | -0.2761 | -1.023 | -0.3397 |
| 2.0/2.5 | -2.000 | -0.7564 | -2.991 | -0.8797 |
| 2.2/2.0 | -0.9000 | -0.3673 | | |
| 2.8/2.0 | -1.552 | -0.4058 | | |

TABLE I: Results for the c_D and c_E couplings fit to $E_{3H} = -8.482$ MeV and $r_{4He} = 1.95 - 1.96$ fm for the NN/3N cutoffs used here. For $V_{\text{low } k}$ (SRG) interactions, the 3NF fits lead to $E_{4He} = -28.22 \dots 28.45$ MeV ($-28.53 \dots 28.71$ MeV).

a fit to data for all cutoffs [14]. In the future, it will be possible to fully evolve three- and four-body forces starting from chiral EFT [18, 19]. The fit values of Table I are natural and the predicted ^4He binding energies are very reasonable. We have also verified that the resulting 3NF becomes perturbative in $A = 3, 4$ (see also Ref. [14]) for the cutoffs used and generally for $\Lambda_{3NF} \lesssim \Lambda$ [15]. The sensitivity of many-body observables to uncertainties in the c_i coefficients was demonstrated recently in neutron matter calculations [20]. This raises the possibility of using nuclear matter to constrain some of the c_i couplings.

The evolution of the cutoff Λ to smaller values is accompanied by a shift of physics. In particular, effects due to iterated tensor interactions, which peak in the relative momentum range $k \sim 4 \text{ fm}^{-1}$ (and thus lead to saturation at too high density), are replaced by three-body contributions. The role of the 3NF for saturation is demonstrated in Fig. 2. The two pairs of curves show the difference between the nuclear matter results for NN-only and NN plus 3N interactions. It is evident that saturation

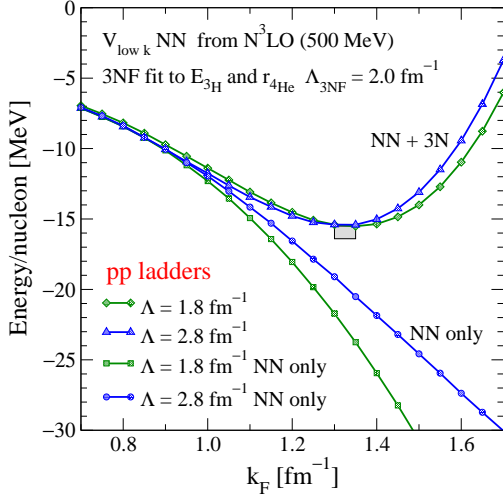


FIG. 2: (Color online) Nuclear matter energy of Fig. 1 at the particle-particle-ladders level compared to NN-only results for two representative NN cutoffs and a fixed 3N cutoff.

is driven by the 3NF [10]. Even for $\Lambda = 2.8 \text{ fm}^{-1}$, which is similar to the lower cutoffs in chiral EFT potentials, saturation is at too high density without the 3NF. Nevertheless, as in previous results [10], the 3N contributions and the c_D, c_E fits are natural, and the same is expected for the ratio of three- to four-body contributions.

The smooth RG evolution used in the results so far is not the only choice for low-momentum interactions. A recent development is the use of flow equations to evolve Hamiltonians, which we refer to as the Similarity Renormalization Group (SRG) [21, 22, 23]. The flow parameter λ , which has dimensions of a momentum, is a measure of the degree of decoupling in momentum space. Few-body results for roughly the same value of SRG λ and smooth $V_{\text{low } k}$ Λ have been remarkably similar (see, for example, Ref. [11]). The analogous nuclear matter energies shown in Fig. 3 are also similar, which helps support the general nature of the 3NF fit. On the other hand, the difference of 2 MeV per particle at saturation and above enlarges our theoretical uncertainty.

The presented results are starting from a chiral EFT potential at a single order with one choice of EFT regulator implementation and values. There are several alternatives available [5, 6, 24], which are largely phase-shift equivalent (the chi-square is not equally good up to $E_{\text{lab}} \approx 300 \text{ MeV}$). Universality for phase-shift equivalent chiral EFT potentials was shown for smooth-cutoff $V_{\text{low } k}$ interactions in Ref. [8] in the form of a collapse of the different potentials to the same matrix elements. An analogous collapse for $N^3\text{LO}$ potentials evolved by the SRG is shown in Fig. 4 for the 1S_0 channel, with a comparison to the corresponding $V_{\text{low } k}$ interactions. Similar results are found in the other channels. Based on this universal collapse, we do not anticipate large differences in nuclear

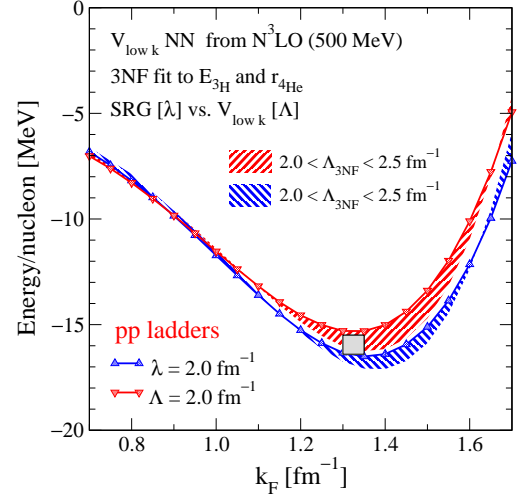


FIG. 3: (Color online) Nuclear matter energy of Fig. 1 at the particle-particle-ladders level comparing low-momentum $V_{\text{low } k}$ with SRG-evolved chiral NN interactions.

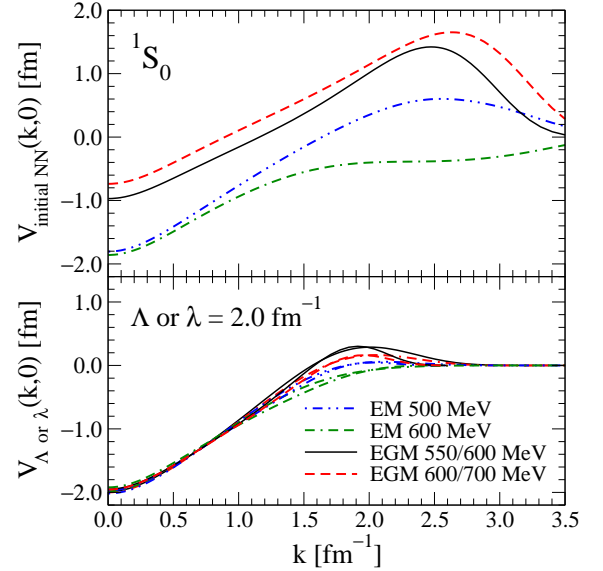


FIG. 4: (Color online) Off-diagonal momentum-space matrix elements of different chiral $N^3\text{LO}$ potentials (EM [5] and EGM [6]) in the 1S_0 channel (upper panel) and after evolution to low-momentum $V_{\text{low } k}$ or SRG interactions (lower panel). A similar universal collapse is found for the low-momentum diagonal matrix elements and in other channels.

matter when starting the evolution with different $N^3\text{LO}$ potentials, but a detailed comparison is forthcoming [15].

The theoretical errors of our results arise from truncations in the initial chiral EFT Hamiltonian, the approximation of the 3NF, and the many-body approximations. We do not claim a chiral expansion for nuclear matter saturation, only that the hierarchy of potential energy contributions is maintained by the RG/SRG evolution. Is

the nuclear matter many-body calculation under control? Corrections to the present approximation include higher-order terms in the hole-line expansion and particle-hole corrections. While we have positive circumstantial evidence from cutoff independence that these corrections are small, we believe that an approach such as coupled cluster theory that can perform a high-level resummation is necessary for a robust validation.

While nuclear matter has lost its status to light nuclei as the first step to nuclear structure, it is still key as a step to heavier nuclei. Our results open the door to ab-initio density functional theory (DFT) based on expanding about nuclear matter [25]. This is analogous to the application of DFT in quantum chemistry and condensed matter starting with the uniform electron gas in local-density approximations and adding constrained derivative corrections. Phenomenological energy functionals (such as Skyrme) for nuclei have impressive successes but lack a (quantitative) microscopic foundation based on nuclear forces and seem to have reached the limits of improvement with the current form of functionals [26, 27]. The theoretical errors of our results, while impressively small on the scale of the potential energy per particle, are far too large to be quantitatively competitive with existing functionals. However, there is the possibility of fine tuning to heavy nuclei, of using EFT/RG to guide next-generation functional forms, and of benchmarking with ab-initio methods for low-momentum interactions. Work in these directions is in progress.

In summary, we have presented the first results for nuclear matter based on chiral NN and 3N interactions with RG evolution. The chiral EFT framework provides a systematic improvable Hamiltonian while the softening of nuclear forces by RG evolution enhances the convergence and control of the many-body calculation. The empirical saturation point is reproduced with theoretical uncertainties despite input only from few-body data. Because of the fine cancellations, however, significant reduction of the errors will be needed before direct DFT calculations of nuclei are competitive. Nevertheless, these results are very promising for a unified description of all nuclei and nuclear matter.

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